An overview of modern monetary theory

(From "Monetary Policy, Inflation, and the Business Cycle", by Jordi Galí, 2008, Princeton University Press)

- This course offers an simple introduction to the basic frame work and a discussion of its policy implications.

- The need for a framework that can help us understand the links between monetary policy and the aggregate performance of an economy seems self-evident:

- On the one hand, citizens of modern societies have good reason to care about developments in inflation, employment, and other economy-wide variables, for those developments affect to an important degree people's opportunities to maintain or improve their standard of living.

- On the other hand, monetary policy, as conducted by central banks, has an important role in shaping those macroeconomic developments, both at the national and supranational levels. Changes in interest rates have a direct effect on the valuation of financial assets and their expected returns, as well as on the consumption and investment decisions of households and firms. Those decisions can in turn have consequences for gross domestic product (GDP) growth, employment, and inflation. It would thus seem important to understand how those interest rate decisions end up affecting the various measures of an economy's performance, both nominal and real. A key goal of monetary theory is to provide us with an account of the mechanisms through which those effects arise, i.e., the transmission mechanism of monetary policy.

- Central banks do not change interest rates in an arbitrary or whimsical manner. Understanding what should be the objectives of monetary policy and how the latter should be conducted in order to attain those objectives constitutes another important aim of modern monetary theory in its normative dimension.

Real Business Cycle (RBC) Theory and Classical Monetary Models

- The impact of the RBC revolution had both a methodological and a conceptual dimension. (Kydland and Prescott (1982)).

- From a methodological point of view, RBC theory firmly established the use of dynamic stochastic general equilibrium (DSGE) models as a central tool for macroeconomic analysis

- From a conceptual point of view:
  i) The efficiency of business cycles $\Rightarrow$ Stabilization policies may not be necessary or desirable.

  ii) The importance of technology shocks as a source of economic fluctuations.

  iii) The limited role of monetary factors

- Introducing a monetary sector in an conventional RBC (under the assumptions of perfect competition and fully flexible prices and wages) were not perceived by central banks and other policy institutions, as yielding a framework that was relevant for policy analysis. In general, the classical monetary model predicts neutrality of monetary policy with respect to real variables.

- The classical monetary models usually yield as a normative implication the optimality of the Friedman rule (a policy that requires central banks to keep the short term nominal rate constant at a zero level)

The New Keynesian Model: Main Elements and Features

- The New Keynesian Model shares with the RBC model important similarities (***microfoundations***): i) an infinitely-lived representative household that seeks to maximize the utility from consumption and leisure, subject to an intertemporal budget constraint, and (ii) a large number of firms with access to an identical technology, subject to exogenous random shifts. Though endogenous capital accumulation, a key element of RBC theory, is absent in canonical versions of the New Keynesian model, it is easy to incorporate and is a common feature of medium-scale versions.6 Also, as in RBC theory, an equilibrium takes the form of a stochastic process for all the economy's endogenous variables consistent with optimal intertemporal decisions by households and firms, given their objectives and constraints and with the clearing of all markets.

- The New Keynesian modelling approach, however, combines the DSGE structure characteristic of RBC models with assumptions that depart from those found in classical monetary models:
  i) Monopolistic competition

  ii) Nominal rigidities

  iii) Short run non neutrality of monetary policy. It is a natural consequence of the presence of nominal rigidities

  Implications:
  i) the economy's response to shocks is generally inefficient.
  ii) the non-neutrality of monetary policy resulting from the presence of nominal rigidities makes room for potentially welfare-enhancing interventions by the monetary authority in order to minimize the existing distortions.
  iii) Furthermore, those models are arguably suited for the analysis and comparison of alternative monetary regimes without being subject to the Lucas critique

- Is the evidence consistent with that prediction of models with nominal rigidities? And if so, are the effects of monetary policy interventions sufficiently important quantitatively to be relevant?
- Unfortunately, identifying the effects of changes in monetary policy is not an easy task. The reason for this is well understood: An important part of the movements in whatever variable is taken as the instrument of monetary policy (e.g., the short term nominal rate) are likely to be endogenous, i.e., the result of a deliberate response of the monetary authority to developments in the economy.
- Thus, simple correlations of interest rates (or the money supply) on output or other real variables cannot be used as evidence of non-neutralities. The direction of causality could well go, fully or in part, from movements in the real variable (resulting from nonmonetary forces) to the monetary variable.
- The main challenge facing that literature lies in identifying changes in policy that could be interpreted as autonomous (Structural (or identified) Vector Autoregressions).

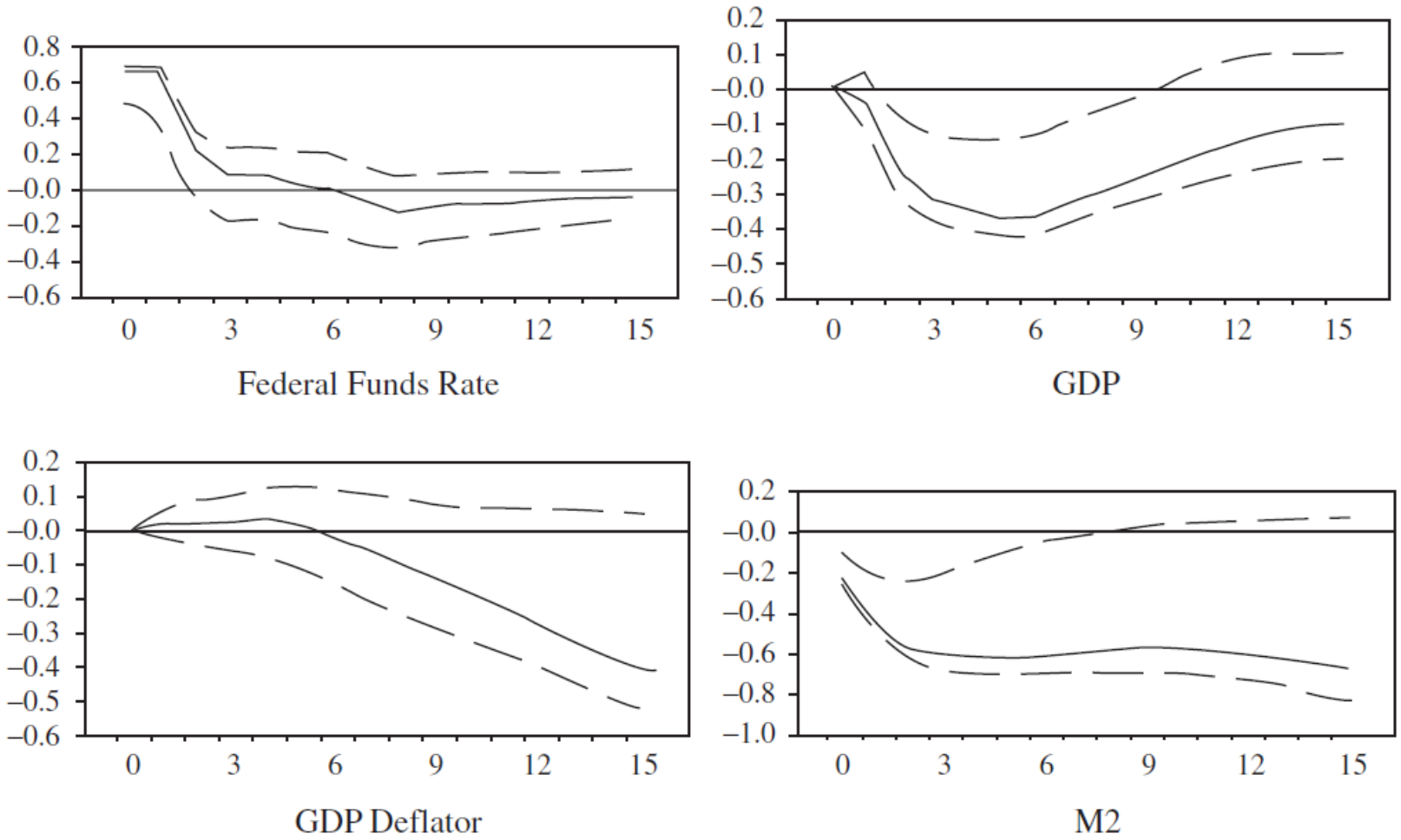

Figure 1.1 Estimated Dynamic Response to a Monetary Policy Shock

Source: Christiano, Eichenbaum, and Evans (1999).

- In response to a tightening of policy, GDP declines with a characteristic hump-shaped pattern. That estimated response of GDP *can be viewed as evidence of sizable and* ***persistent real effects of monetary policy shocks***.

- The (log) GDP deflator displays a flat response for over a year, after which it declines. That estimated sluggish response of prices to the policy tightening is generally interpreted as evidence of ***substantial price rigidities***.

- Finally, note that (log) M2 displays a persistent decline in the face of the rise in the federal funds rate, suggesting that the Fed needs to reduce the amount of money in circulation in order to bring about the increase in the nominal rate. The observed negative comovement between money supply and nominal interest rates is known as ***liquidity effect***.

A Classical Monetary Model

(From “Monetary Policy, Inflation, and the Business Cycle”, by Jordi Galí, 2008, Princeton University Press, chapter 2)

A version of the slides by Jordi Galí in

http://www.crei.cat/people/gali/teaching_gali/teaching_pagina.html#content

Assumptions

- Perfect competition in goods and labor markets
- Flexible prices and wages
- No capital accumulation
- No fiscal sector
- Closed economy

Outline

- The problem of households and firms
- Equilibrium: monetary policy neutrality
- Monetary policy and the determination of nominal variables

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{1}$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \tag{2}$$

for $t = 0, 1, 2, ...$and the solvency constraint

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} (B_T / P_T) \right\} \geq 0 \tag{3}$$

where $\Lambda_{t,T} \equiv \beta^{T-t} U_{c,T} / U_{c,t}$ is the stochastic discount factor.

Optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{4}$$

$$Q_t = \beta \; E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \; \frac{P_t}{P_{t+1}} \right\} \tag{5}$$

Specification of utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

implied optimality conditions:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \tag{6}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

Log-linear versions

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{8}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{9}$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$ (interpretation)

Perfect foresight steady state (with zero growth):

$$i = \pi + \rho$$

hence implying a real rate

$$r \equiv i - \pi = \rho$$

Ad-hoc money demand

$$m_t - p_t = c_t - \eta i_t$$

Firms

Representative firm with technology

$$Y_t = A_t N_t^{1-\alpha} \tag{10}$$

Profit maximization:

$$\max \; P_t Y_t - W_t N_t$$

subject to (10), taking the price and wage as given (perfect competition)

Optimality condition:

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

In log-linear terms

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha) \tag{11}$$

Equilibrium

Goods market clearing

$$y_t = c_t \tag{12}$$

Labor market clearing

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1-\alpha)$$

Asset market clearing:

$$B_t = 0$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate output:

$$y_t = a_t + (1-\alpha)n_t$$

Implied equilibrium values for real variables:

$$
\begin{aligned}
n_t &= \psi_{na} a_t + \vartheta_n \\
y_t &= \psi_{ya} a_t + \vartheta_y \\
r_t &\equiv i_t - E_t\{\pi_{t+1}\} \\
&= \rho + \sigma\ E_t\{\Delta y_{t+1}\} \\
&= \rho + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} \\
\omega_t &\equiv w_t - p_t \\
&= a_t - \alpha n_t + \log(1-\alpha) \\
&= \psi_{\omega a} a_t + \log(1-\alpha)
\end{aligned}
$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$; $\vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$;
$\vartheta_y \equiv (1-\alpha)\vartheta_n$; $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$

$\Longrightarrow$ *neutrality*: real variables determined *independently of monetary policy*

$\Longrightarrow$ *optimal policy*: undetermined.

$\Longrightarrow$ specification of monetary policy needed to determine *nominal* variables

Monetary Policy and Price Level Determination

Example I: An Exogenous Path for the Nominal Interest Rate

$$i_t = i + v_t$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Particular case: $i_t = i$ for all t.

Implied steady state inflation: $\pi = i - \rho$

Using definition of real rate:

$$\begin{aligned} E_t\{\pi_{t+1}\} &= i_t - r_t \\ &= \pi + v_t - \widehat{r}_t \end{aligned}$$

Equilibrium inflation:

$$\pi_t = \pi + v_{t-1} - \widehat{r}_{t-1} + \xi_t$$

with $E_t\{\xi_{t+1}\} = 0$ for all t.

Implied path for the money supply:

$$m_t = p_t + y_t - \eta i_t$$

Example II: A Simple Interest Rate Rule

$$i_t = \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t$$

where $\phi_\pi \geq 0$. Combined with definition of real rate:

$$\phi_\pi \widehat{\pi}_t = E_t\{\widehat{\pi}_{t+1}\} + \widehat{r}_t - v_t$$

If $\phi_\pi > 1$,

$$\begin{aligned} \widehat{\pi}_t &= \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\widehat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a}\, a_t - \frac{1}{\phi_\pi - \rho_v}\, v_t \end{aligned}$$

If $\phi_\pi < 1$,

$$\widehat{\pi}_t = \phi_\pi \widehat{\pi}_{t-1} - \widehat{r}_{t-1} + v_{t-1} + \xi_t$$

where $E_t\{\xi_{t+1}\} = 0$ for all t

$\Longrightarrow$ *price level indeterminacy*

$\Longrightarrow$ illustration of the "Taylor principle" requirement

Example III: An Exogenous Path for the Money Supply $\{m_t\}$

Combining money demand and the definition of the real rate:

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$. Solving forward:

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\left\{m_{t+k}\right\} + \overline{u}_t$$

where $\overline{u}_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{u_{t+k}\}$

$\Rightarrow$ *price level determinacy*

In terms of money growth rates:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \left\{ \Delta m_{t+k} \right\} + \overline{u}_t$$

Nominal interest rate:

$$\begin{aligned} i_t &= \eta^{-1} \left(y_t - (m_t - p_t) \right) \\ &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \left\{ \Delta m_{t+k} \right\} + z_t \end{aligned}$$

where $z_t \equiv \eta^{-1}(\overline{u}_t + y_t)$ is independent of monetary policy.

Example

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Assume no real shocks ($r_t = y_t = 0$).

Price response:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$$\Longrightarrow \textit{large price response}$$

Nominal interest rate response:

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$$\Longrightarrow \textit{no liquidity effect}$$

A Model with Money in the Utility Function

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

Budget constraint

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t$$

with solvency constraint:

$$\lim_{T\to\infty} E_t \left\{\Lambda_{t,T}(\mathcal{A}_T/P_T)\right\} \geq 0$$

where $\mathcal{A}_t \equiv B_t + M_t$.

Equivalently:

$$P_t C_t + Q_t \mathcal{A}_t + (1 - Q_t) M_t \leq \mathcal{A}_{t-1} + W_t N_t + D_t$$

Interpretation: $(1 - Q_t) = 1 - \exp\{-i_t\} \simeq i_t$

$\Longrightarrow$ opportunity cost of holding money

Optimality Conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

Two cases:

- utility separable in real balances $\Longrightarrow$ neutrality
- utility non-separable in real balances $\Longrightarrow$ non-neutrality

Utility specification:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{Z(C_t, M_t/P_t)^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$\begin{aligned} Z(C_t, M_t/P_t) &\equiv \left((1-\vartheta)C_t^{1-\nu} + \vartheta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right)^{\frac{1}{1-v}} \qquad for\ \nu \neq 1 \\ &\equiv C_t^{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{\vartheta} \qquad for\ \nu = 1 \end{aligned}$$

Note that

$$\begin{aligned} U_{c,t} &= (1-\vartheta)Z_t^{\nu-\sigma}C_t^{-\nu} \\ U_{m,t} &= \vartheta Z_t^{\nu-\sigma}\left(M_t/P_t\right)^{-\nu} \\ U_{n,t} &= -N_t^{\varphi} \end{aligned}$$

Implied optimality conditions:

$$W_t/P_t = N_t^{\varphi} Z_t^{\sigma-\nu} C_t^{\nu} (1-\vartheta)^{-1}$$

$$Q_t = \beta E_t \left\{ (C_{t+1}/C_t)^{-\nu} (Z_{t+1}/Z_t)^{\nu-\sigma} (P_t/P_{t+1}) \right\}$$

$$M_t/P_t = C_t \left(1 - \exp\{-i_t\}\right)^{-\frac{1}{\nu}} \left(\vartheta/(1-\vartheta)\right)^{\frac{1}{\nu}}$$

Log-linearized money demand equation:

$$m_t - p_t = c_t - \eta i_t$$

where $\eta \equiv 1/[\nu(\exp\{i\} - 1)]$

Log-linearized labor supply equation

$$w_t - p_t = \sigma c_t + \varphi n_t + \varpi i_t$$

where $\varpi \equiv \frac{k_m \beta (1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)}$ and $k_m \equiv \frac{M/P}{C}$

Discussion

Equilibrium

Labor market clearing:

$$\sigma c_t + \varphi n_t + \varpi i_t = a_t - \alpha n_t + \log(1-\alpha)$$

which combined with aggregate production function:

$$y_t = \psi_{ya} a_t + \psi_{yi} i_t$$

where $\psi_{yi} \equiv -\frac{\varpi(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

Assessment of size of non-neutralities

Calibration: $\sigma = \varphi = 1$; $\alpha = 1/3$; $\nu = 1/\eta i$ "large"

$$\Rightarrow \varpi \simeq \frac{k_m \beta}{1 + k_m(1-\beta)} > 0 \quad ; \quad \psi_{yi} \simeq -\frac{k_m}{3} < 0$$

Monetary base inverse velocity: $k_m \simeq 0.3 \qquad \Rightarrow \psi_{yi} \simeq -0.1$

M2 inverse velocity: $k_m \simeq 3 \qquad \Rightarrow \psi_{yi} \simeq -1$

$\Rightarrow$ small output effects of monetary policy

$\Rightarrow$ counterfactual comovements in response to monetary shocks ($\uparrow i, \downarrow y \Leftrightarrow \uparrow \pi, \uparrow \Delta m$)

Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

Social Planner's problem

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to

$$C_t = A_t N_t^{1-\alpha}$$

Optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) A_t N_t^{-\alpha} \tag{13}$$

$$U_{m,t} = 0 \tag{14}$$

Optimal policy (Friedman rule): $i_t = 0$ for all t .

Intuition

Implied average inflation: $\pi = -\rho < 0$

Implementation

$$i_t = \phi(r_{t-1} + \pi_t)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi i_t$$

whose only stationary solution is $i_t = 0$ for all t.

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$

The Basic New Keynesian Model

(From "Monetary Policy, Inflation, and the Business Cycle", by Jordi Galí, 2008, Princeton University Press, chapter 3)

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Motivation and Outline

Evidence on Money, Output, and Prices:

- Short Run Effects of Monetary Policy Shocks

  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

- Micro Evidence on Price-setting Behavior: significant price and wage rigidities

A Baseline Model with Nominal Rigidities

- monopolistic competition
- sticky prices (staggered price setting)
- competitive labor markets, closed economy, no capital accumulation

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

where

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, ...$ plus solvency constraint

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} \frac{B_T}{P_T} \right\} \geq 0$$

Optimality conditions

1. Optimal allocation of expenditures

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

implying

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t$$

where

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

2. Other optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}\right\}$$

Specification of utility:

$$U(C_t, N_t; X_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) X_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) X_t & \text{for } \sigma = 1 \end{cases}$$

where

$$x_t = \rho_x x_{t-1} + \varepsilon_t^x$$

Optimality conditions:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_x)x_t$$

Ad-hoc money demand:

$$m_t - p_t = y_t - \eta i_t$$

Firms

- Continuum of firms, indexed by $i \in [0, 1]$
- Each firm produces a differentiated good
- Identical technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

  where

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- Probability of resetting price in any given period: $1 - \theta$, independent across firms (Calvo (1983)).
- $\theta \in [0, 1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$

Aggregate Price Dynamics

$$P_t = \left[\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Dividing by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

Log-linearization around zero inflation steady state

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{1}$$

or, equivalently

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

Optimal Price Setting

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{2}$$

for $k = 0, 1, 2, ...$where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0 \tag{3}$$

where $\Psi_{t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{t+k|t})$ and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$.

Flexible price case ($\theta = 0$):

$$P_t^* = \mathcal{M} \Psi_{t|t}$$

Alternative representation:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} - \frac{\mathcal{M}}{\mathcal{M}_{t+k}} \frac{\Psi_{t+k|t}}{\Psi_{t+k}} \right) \right\} = 0 \tag{4}$$

where $\mathcal{M}_t \equiv P_t/\Psi_t$ and $\Psi_t \equiv \left(\int_0^1 \Psi_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$

Zero inflation steady state

$$\Lambda_{t,t+k} = \beta^k \ ; \ P_t^*/P_{t-k} = P_t/P_{t-k} = 1 \ ; \ Y_{t+k|t} = Y \ ; \ \Psi_{t+k|t} = \Psi_t \ ; \ P_t = \mathcal{M}\Psi_t$$

Linearized optimal price setting condition:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k|t}\}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ and $\mu \equiv \log \mathcal{M}$

Particular Case: $\alpha = 0$ (constant returns)

$$\Longrightarrow \psi_{t+k|t} = \psi_{t+k}$$

Recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)p_t - (1-\beta\theta)\widehat{\mu}_t$$

Combined with price dynamics equation:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

General case: $\alpha \in [0,1)$

$$\begin{aligned}\psi_{t+k|t} &= w_{t+k} - mpn_{t+k|t} \\ &= w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \log(1-\alpha))\end{aligned}$$

$$\psi_{t+k} = w_{t+k} - (a_{t+k} - \alpha n_{t+k} + \log(1-\alpha))$$

$$\begin{aligned}\psi_{t+k|t} &= \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\ &= \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k})\end{aligned}$$

Optimal price setting equation:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k} - \Theta\widehat{\mu}_{t+k}\right\}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \in (0,1]$.

Recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)p_t - (1-\beta\theta)\Theta\widehat{\mu}_t$$

Combined with price dynamics equation:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\Theta$$

Equilibrium

Goods markets clearing

$$Y_t(i) = C_t(i)$$

for all $i \in [0, 1]$ and all t.

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$:

$$Y_t = C_t$$

Combined with Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_x)x_t$$

Labor market clearing:

$$\begin{aligned} N_t &= \int_0^1 N_t(i)di \\ &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \end{aligned}$$

Up to a first order approximation:

$$n_t = \frac{1}{1-\alpha}(y_t - a_t)$$

Average price markup and output

$$\begin{aligned}
\mu_t &= p_t - \psi_t \\
&= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}$$

Under flexible prices:

$$\mu = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)$$

Thus,

$$\widehat{\mu}_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

The Non-Policy Block of the Basic New Keynesian Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

where r_t^n is the *natural rate of interest*, given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_x)x_t$$

Missing block: description of monetary policy (determination of i_t).

Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

where $\widehat{y}_t \equiv y_t - y$ and

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Equivalently:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n + v_t$$

where $\widehat{y}_t^n \equiv y_t^n - y$.

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \; u_t$$

where

$$\begin{aligned} u_t &\equiv \widehat{r}_t^n - \phi_y \widehat{y}_t^n - v_t \\ &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t + (1-\rho_x)x_t - v_t \end{aligned}$$

and

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma+\phi_y) \end{bmatrix} \qquad ; \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

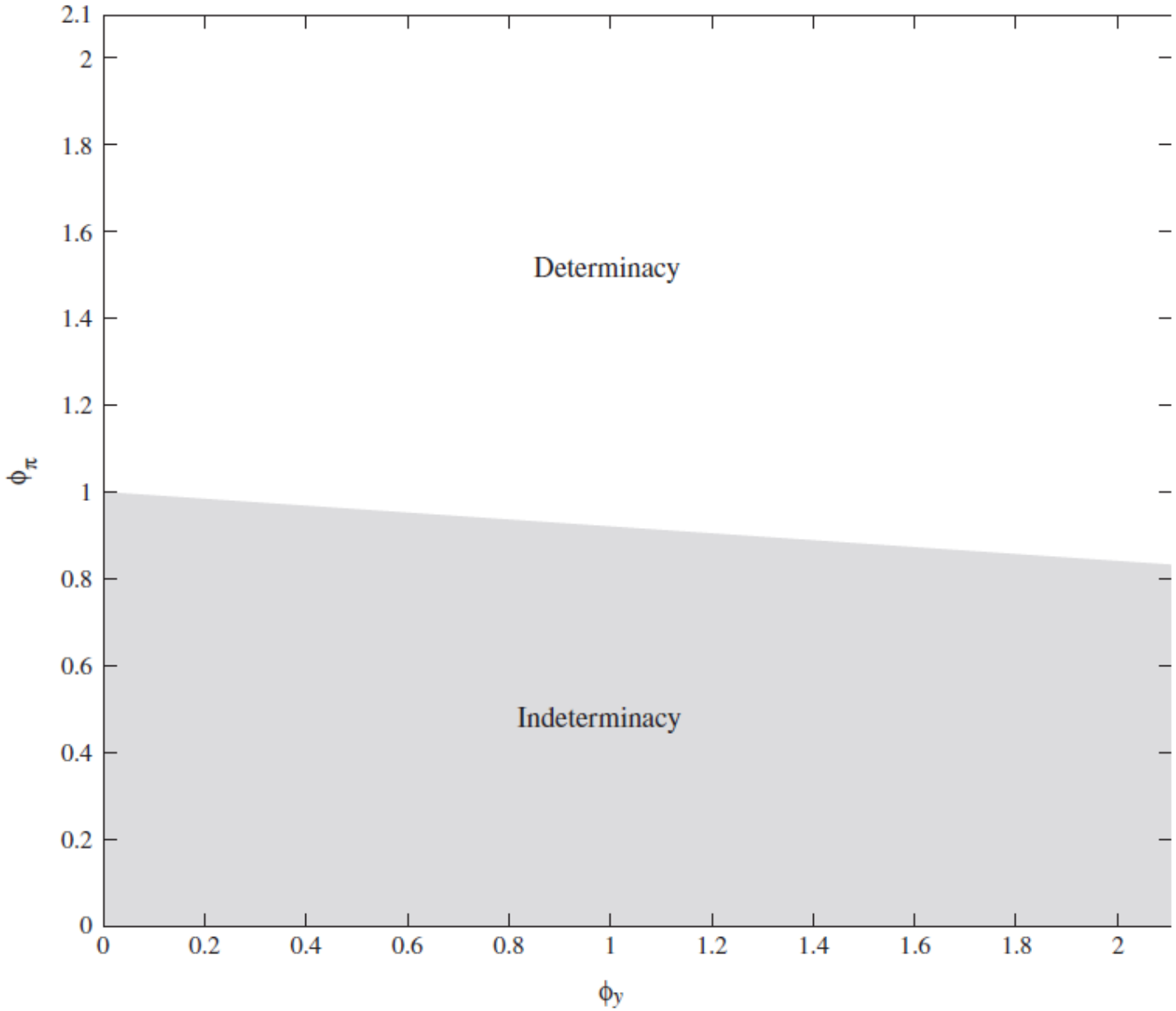
with $\Omega \equiv \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}$.

Uniqueness condition (Bullard and Mitra):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

Exercise: analytical solution (method of undetermined coefficients).

Equilibrium uniqueness under the simple interest rate rule



Equilibrium under an Exogenous Money Growth Process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Money demand

$$l_t \equiv m_t - p_t = \widetilde{y}_t - \eta i_t + y_t^n$$

Substituting into dynamic IS equation

$$(1 + \sigma\eta)\,\widetilde{y}_t = \sigma\eta E_t\{\widetilde{y}_{t+1}\} + \widehat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta\widehat{r}_t^n - \widehat{y}_t^n$$

Identity:

$$\widehat{l}_{t-1} = \widehat{l}_t + \pi_t - \Delta m_t$$

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta m_t \end{bmatrix} \tag{5}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Uniqueness condition:

$\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}}\mathbf{A_{M,1}}$ has two eigenvalues inside and one outside the unit circle.

Calibration

Households: $\sigma = \varphi = 1$; $\beta = 0.99$; $\epsilon = 6$; $\eta = 4$; $\rho_v = 0.5$

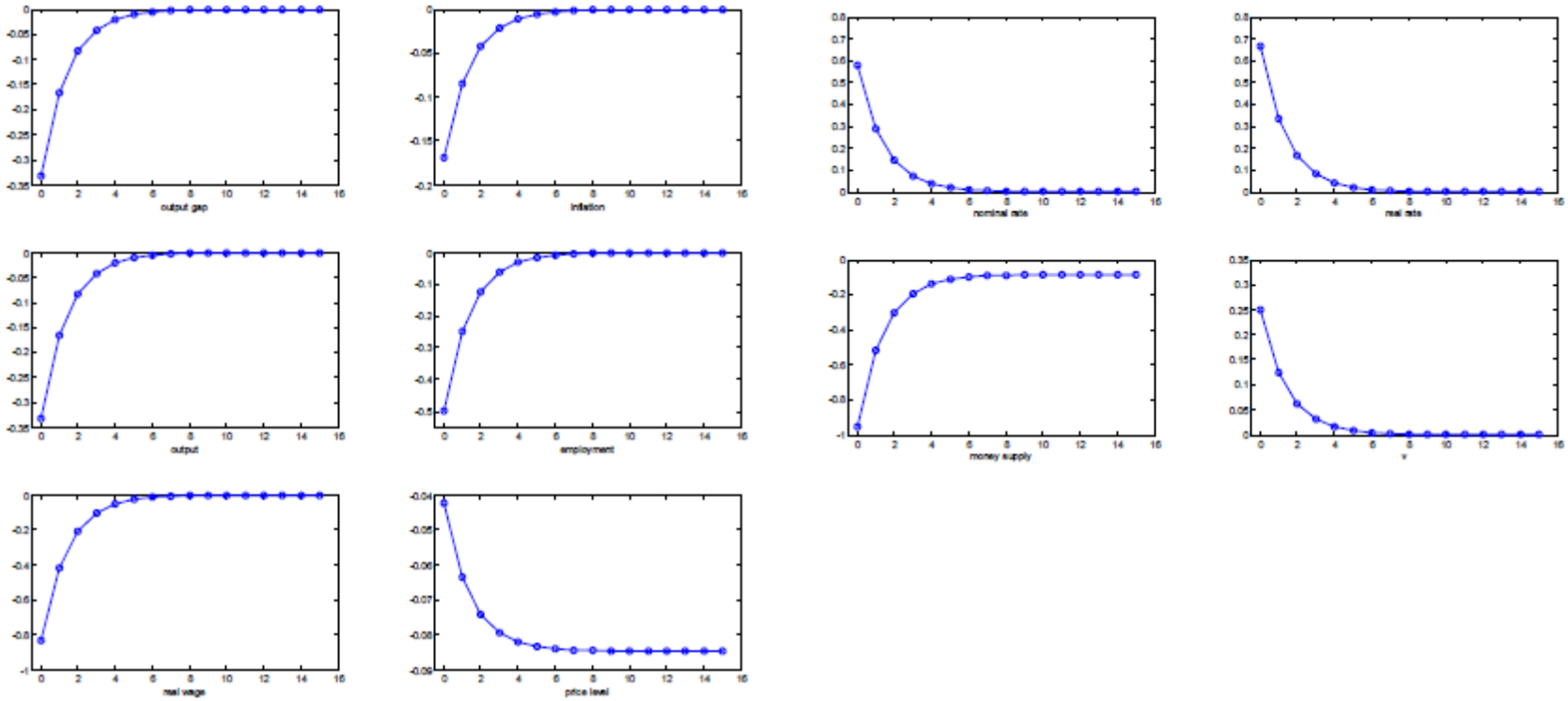
Firms: $\alpha = 1/3$; $\theta = 3/4$; $\rho_a = 0.9$

Policy rules: $\phi_\pi = 1.5$, $\phi_y = 0.125$; $\rho_v = \rho_m = 0.5$

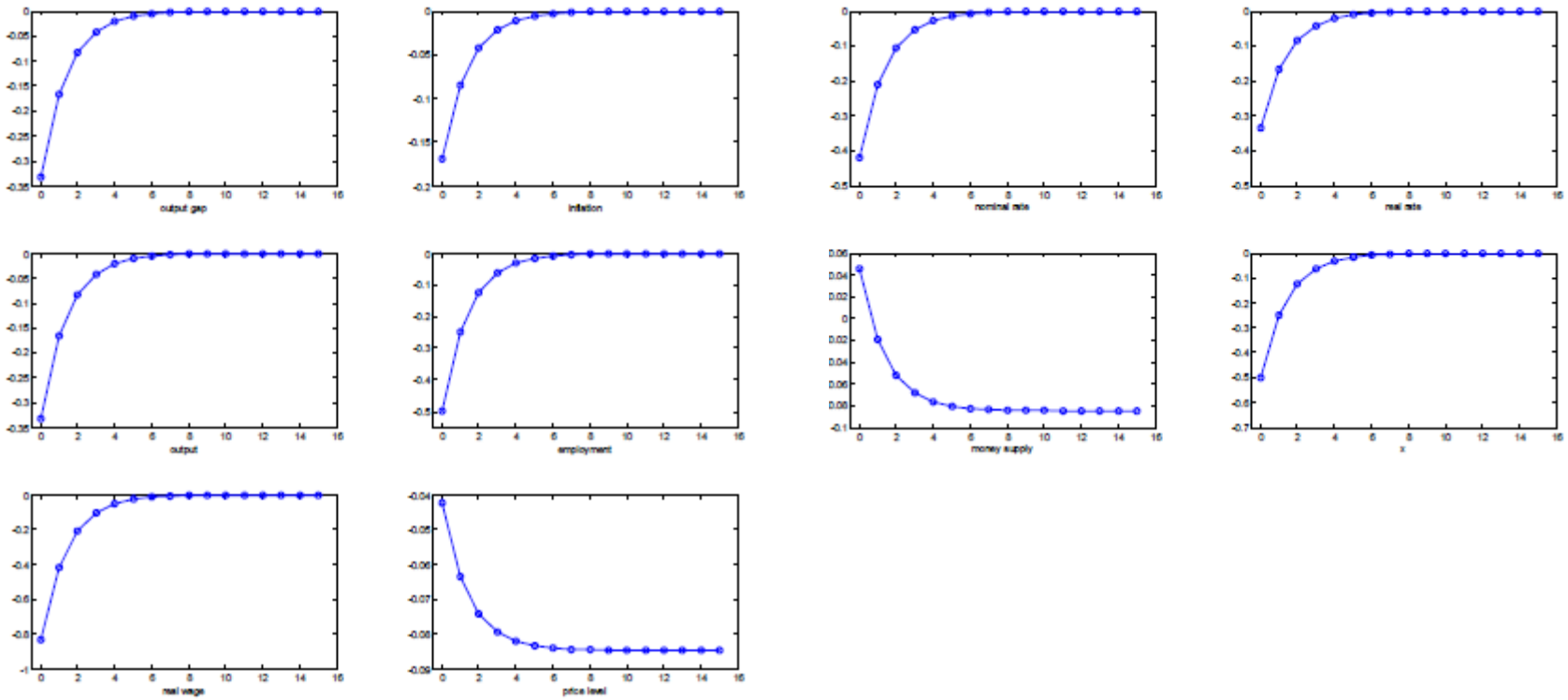
Dynamic Responses to Exogenous Shocks

- Monetary policy, discount rate, technology
- Interest rate rule vs. money growth rule

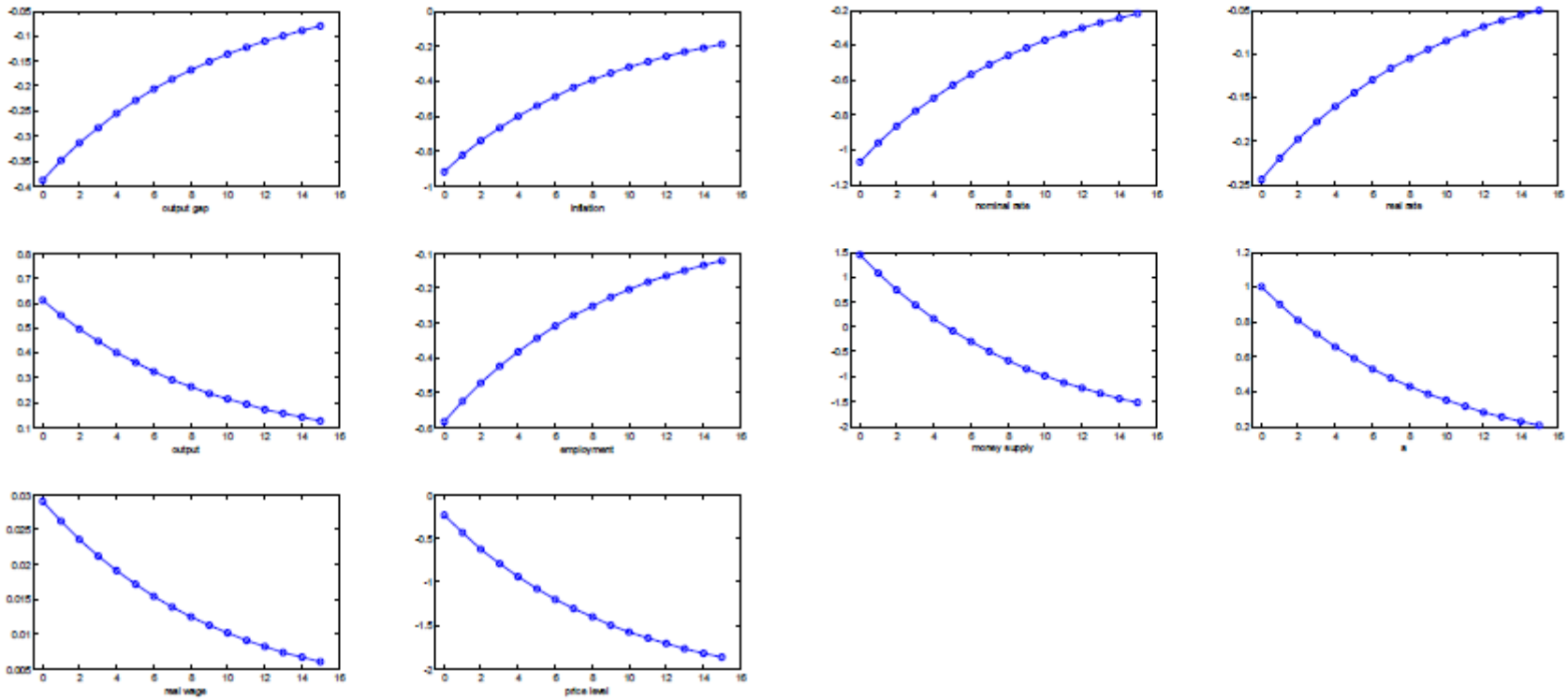
Dynamic responses to a monetary policy shock: Interest rate rule



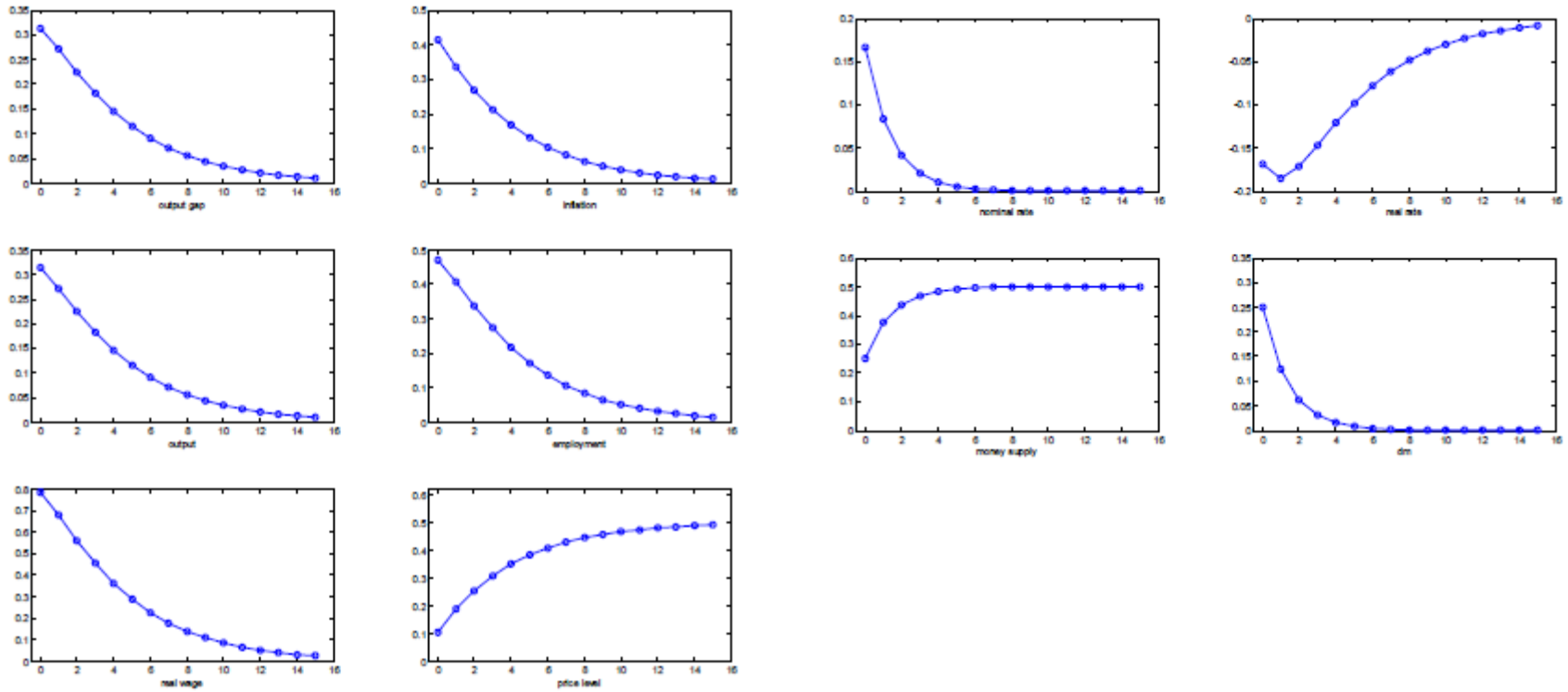
Dynamic responses to a discount rate shock: Interest rate rule



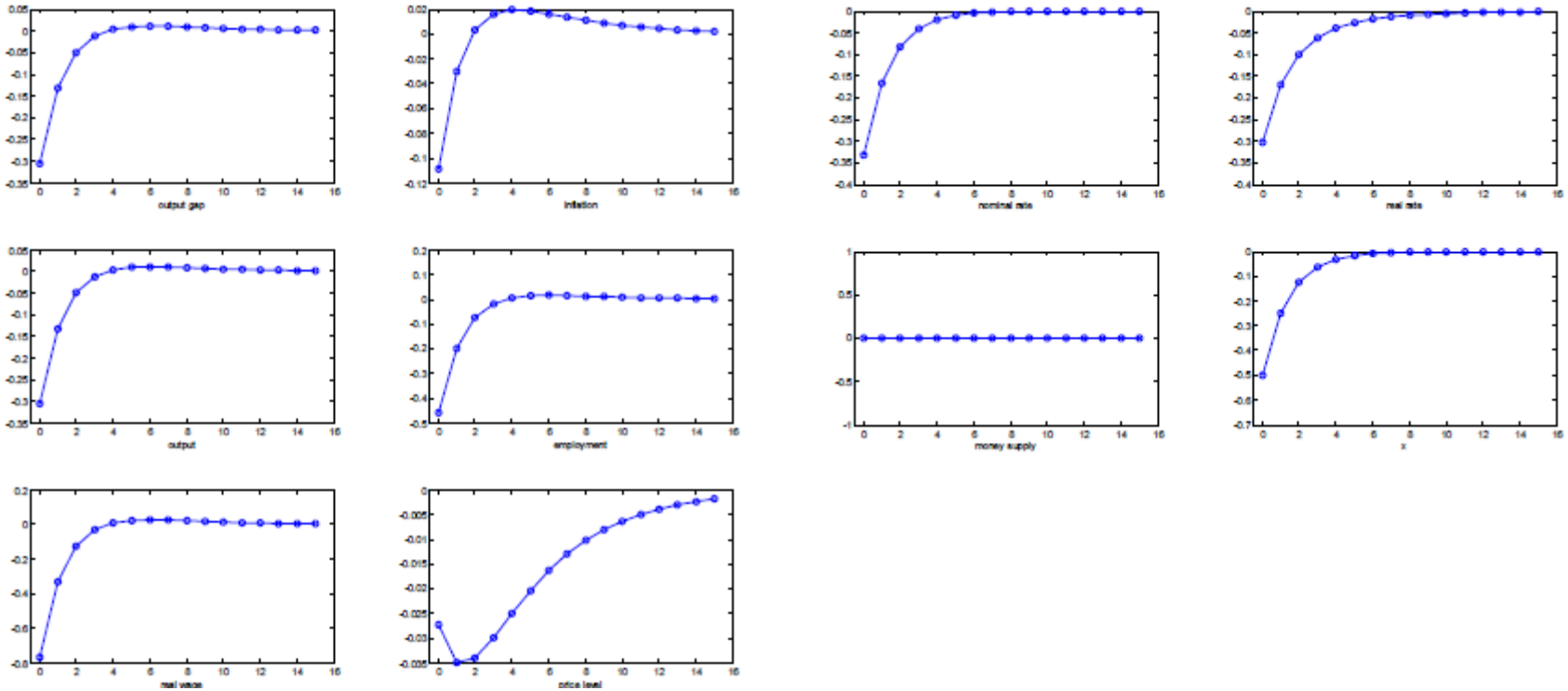
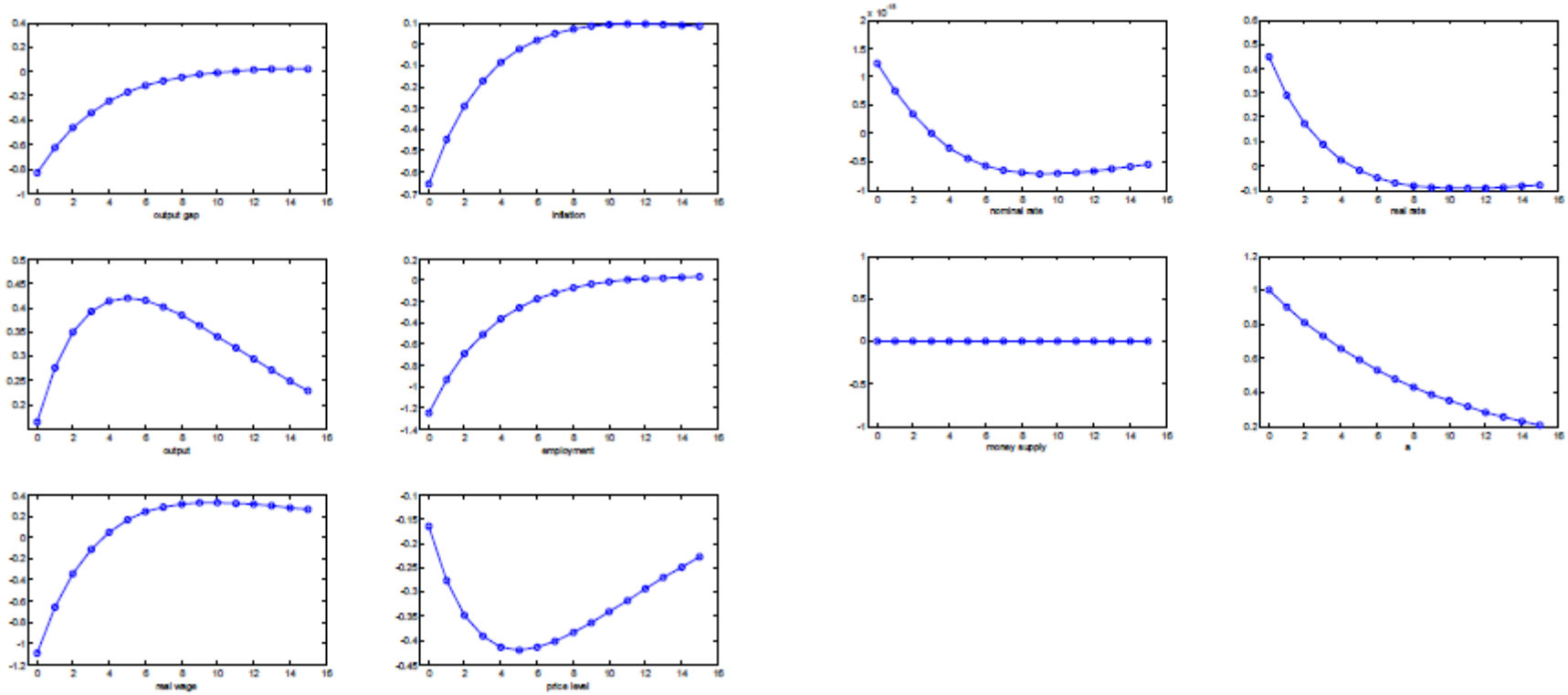
Dynamic responses to a technology shock: Interest rate rule



Dynamic responses to a monetary policy shock: Money growth rule



Dynamic responses to a discount rate shock: Money growth rule



Dynamic responses to a technology shock: Money growth rule